Winning strategy for the "First to call 50" game

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Problem Statement

Two players take turns calling out integers. The first person to call out "50" wins. The rules are:

1. The first player must call an integer between 1 and 10, inclusive
2. Each subsequent number must exceed the previous number by at least 1 and at most 10

Question: Should you go first, and what is your optimal strategy?

Analysis

Let C_i denote the i-th call in the game, where $i = 1, 2, \ldots, w$ and $C_w = 50$ is the winning call.

The game constraints are:

$$1 \leq C_1 \leq 10 \quad \text{(starting condition)} \tag{1}$$
$$C_{i-1} + 1 \leq C_i \leq C_{i-1} + 10 \quad \text{(transition rule)} \tag{2}$$
$$C_w = 50 \quad \text{(winning condition)} \tag{3}$$

From the transition rule, we have $1 \leq C_i - C_{i-1} \leq 10$ for all valid moves.

Backward Analysis

Working backwards from the winning condition:

Final move: To call 50, the opponent's previous call C_{w-1} must satisfy:

$$40 \leq C_{w-1} \leq 49$$

Penultimate move: To force the opponent into the range $[40, 49]$, I need:

$$C_{w-2} = 39$$

This is because: - If $C_{w-2} = 39$, then the opponent must choose from $[40, 49]$ - Whatever the opponent chooses in $[40, 49]$, I can always reach 50 in my next move

General pattern: Continuing this backward analysis, the key insight is that certain positions are *winning positions* - positions from which the current player can force a win with optimal play.

Solution

Optimal Strategy

Go first and call 6. Then, on each subsequent turn, add 11 to your previous call until you reach 50.

Winning Sequence

The optimal play sequence is shown in Table 1:

Turn	Player	My Call	Opponent's Range	Pattern
1	Me	6	—	$6 = 50 - 4 \times 11$
2	Opponent	—	$[7, 16]$	—
3	Me	17	—	$17 = 6 + 11$
4	Opponent	—	$[18, 27]$	—
5	Me	28	—	$28 = 17 + 11$
6	Opponent	—	$[29, 38]$	—
7	Me	39	—	$39 = 28 + 11$
8	Opponent	—	$[40, 49]$	—
9	Me	50	—	$50 = 39 + 11$

Table 1: Optimal game sequence with winning strategy

Why This Works

The strategy exploits the mathematical structure of the game:

1. The difference between any two consecutive calls is between 1 and 10
2. This means each player controls a range of exactly 10 consecutive integers
3. The key insight: $50 \equiv 6 \pmod{11}$
4. By calling numbers of the form $6 + 11k$, I ensure that whatever my opponent calls, I can always reach the next number in my sequence

Mathematical justification: If I call $6 + 11k$, my opponent must call from the range $[6+11k+1, 6+11k+10]$. Since this range has exactly 10 numbers and none are congruent to $6 \pmod{11}$, I can always call $6 + 11(k + 1)$ on my next turn.

Conclusion

This game has a determined winner with perfect play. The first player wins by following the strategy of calling 6 initially and then maintaining the arithmetic progression $6, 17, 28, 39, 50$. The key insight is recognizing that positions congruent to $6 \pmod{11}$ are winning positions, and the starting position allows the first player to seize this advantage immediately.